STOCHASTIC NAVIER-STOKES EQUATIONS: ideas and results using nonstandard analysis

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The **stochastic Navier-Stokes equations** in a bounded domain $D \subset \mathbb{R}^d$ $(d = 2, 3)$ model the velocity of an incompressible viscous fluid confined to the region D and influenced by external non-random and random forces f and gdw:.

$$\begin{cases} du = \big[\nu\Delta u - \langle u, \nabla\rangle u + f(t,u) - \nabla p\big]dt + g(t,u)dw_t \\ \operatorname{div} u = 0 \end{cases}$$

$u(t, x, \omega)$ =(random) velocity of the fluid at the location $x \in D$ at time t:

$$u : [0, \infty) \times D \times \Omega \to \mathbb{R}^d$$

where Ω = domain of an underlying probability space. The initial condition (velocity field) $u(0) = u_0$ may be random

In this talk we will outline the problems associated with these equations (including existence of solutions) and survey a range of ideas and results obtainded using techniques from nonstandard analysis and in particular Loeb measures (measures constructed using nonstandard analysis).

We will not assume any prior knowledge of either nonstandard analysis or fluid mechanics